

# NONLINEAR SPECTRAL MIXING THEORY TO MODEL MULTISPECTRAL SIGNATURES

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## ABSTRACT

Nonlinear spectral mixing occurs due to multiple reflections and transmissions between discrete surfaces, e.g., leaves or facets of a rough surface. The radiosity method is an energy conserving computational method used in thermal engineering and it models nonlinear spectral mixing realistically and accurately. In contrast to the radiative transfer method the radiosity method takes into account the discreteness of the scattering surfaces (e.g., exact location, orientation and shape) such as leaves and includes mutual shading between them.

An analytic radiosity-based scattering model for vegetation was developed and used to compute vegetation indices for various configurations. The leaf reflectance and transmittance was modeled using the PROSPECT model for various amounts of water, chlorophyll and variable leaf structure. The soil background was modeled using SOILSPEC with a linear mixture of reflectances of sand, clay and peat. A neural network and a geometry based retrieval scheme were used to retrieve leaf area

index and chlorophyll concentration for dense canopies. Only simulated canopy reflectances in the 6 visible through short wave IR Landsat TM channels were used. We used a criterion to compute the signal to noise ratio of a retrieved quantity.

## Introduction

### Model Properties:

- Radiosity model of single and N-layer canopy of horizontal leaves
- Leaf reflectance depends on water content, chlorophyll concentration and leaf structure
- Background are random mixtures of sand, clay and peat.

### Datasets:

- Leaf reflectance and transmittance model PROSPECT (Jacquemoud and Baret (1990))
- Soil spectra from SOILSPEC (Jacquemoud et al (1992))

### Spectral Channels:

6 Landsat channels from 0.45  $\mu$  to 2.35  $\mu$ .

### Goals:

- Investigate various commonly used vegetation indices
- Retrieve leaf area index (LAI)
- Retrieve canopy chemistry:
  - Leaf water content:  $C_w$
  - Chlorophyll concentration  $C_{chl}$
  - Leaf structural parameter  $N$

## The Canopy Radiosity Models

### Definition of Radiosity:

Radiosity is the amount of emitted, reflected and transmitted energy per unit time and area leaving the leaf and ground surfaces.

### Assumptions:

- All surfaces are Lambertian reflectors/transmitters
- Canopy consists of a single layer or N-layers with horizontal leaves

## Review of the Basics of the Radiosity Method

The radiosity equation:

$$B_i S_i = E_i S_i + \chi_i \sum_{j=1}^{2N} B_j F_{ji} S_j, \quad i = 1, 2, \dots, 2N, \quad (1)$$

where

$$\chi_i = \begin{cases} \rho_e & \text{if } (\vec{n}_i \cdot \vec{r}_i) < 0 \\ \tau_i & \text{if } (\vec{n}_i \cdot \vec{r}_i) > 0 \end{cases}$$

$B_i$ : Radiosity of the finite area  $S_i$ : the sum of emitted, reflected and transmitted radiative energy per unit time and area leaving a surface  $i$ , unit:  $[Wm^{-2}]$ .

$E_i$ : Emission of the finite area  $S_i$ : the radiative energy per unit time and area emitted from a surface source, e.g., a light source within or on the surface, unit:  $[Wm^{-2}]$ .

$\rho_i$ : Hemispherical reflectance of the finite area  $S_i$ : the fraction of the hemispherically incident radiative flux which is reflected back into the top hemisphere surrounding surface  $i$ , unitless.

$\tau_i$ : Hemispherical transmittance of the finite area  $S_i$ : the fraction of the hemispherically incident radiative flux onto the bottom surface which is transmitted through the surface into the hemisphere surrounding the top of surface  $i$ , unitless.

$\vec{n}_i$ : The normal vector on a surface  $i$  pointing outward.

$F_{ji}$ : View factor or form factor: the fraction of radiative energy leaving the finite surface  $S_j$  that reaches the finite surface element  $S_i$ , sometimes the notation  $F_{S_j \rightarrow S_i}$  is used, unitless.

$N$ : Number of discrete surfaces (e.g., plant leaves), where  $2N$  is the number of (single sided) surface components  $S_i$ .

The radiosity method assumes that:

1. The angular distributions of the radiances leaving the participating surfaces are Lambertian, i.e. constant in all directions.
2. For finite surfaces the magnitude of the emitted radiant flux density (radiosity) does not vary across the respective surfaces.

Using these assumptions and a reciprocity relationship, eq. (1) can be rewritten as:

$$B_i = E_i + \chi_i \sum_{j=1}^{2N} B_j F_{ji}, \quad i = 1, 2, \dots, 2N. \quad (2)$$

Eq. (2) can be solved using the Gauss-Seidel method.

## Review of the Single-Layer Radiosity Model

### Radiosity equations:

$$\begin{aligned} B_1 &= \rho_{lai} E_0 + \tau_{lai} B_3 \\ B_2 &= \tau_{lai} E_0 + \rho_{lai} B_3 \\ B_3 &= \rho_s (1 - lai) E_0 + \rho_s B_2, \end{aligned} \quad (3)$$

where

$E_0$  is the total incident solar power per unit area in  $[Wm^{-2}]$ ,

$lai$  is the leaf area index of a leaf layer without overlapping leaves in  $[m^2m^{-2}]$ ,

$\rho$  and  $\tau$  are the hemispherical reflectance and transmittance of the leaves, and

$\rho_s$  is the soil reflectance.

### Analytical solution:

$$\begin{aligned} B_1 &= E_0 lai \rho + E_0 lai \tau \rho_s \frac{1 + lai(\tau - 1)}{1 - \rho \rho_s lai} \\ B_2 &= E_0 lai \left[ \frac{\tau + \rho \rho_s (1 - lai)}{1 - \rho \rho_s lai} \right] \\ B_3 &= E_0 \rho_s \left[ \frac{1 + lai(\tau - 1)}{1 - \rho \rho_s lai} \right]. \end{aligned} \quad (4)$$

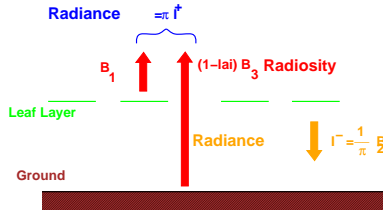


Figure 1: Radiosity and Radiances ( $F^+$ ;  $F^-$ ) for a single canopy layer model above ground.

### BRDF of a single layer canopy away from the hotspot direction:

$$BRDF = \frac{1}{E_0} [B_1 + (1 - lai)B_3] = \rho_{lai} + \rho_s \frac{(1 + (\tau - 1)lai)^2}{1 - \rho \rho_s lai}. \quad (5)$$

## Generation of a Canopy Multi-Spectral Dataset

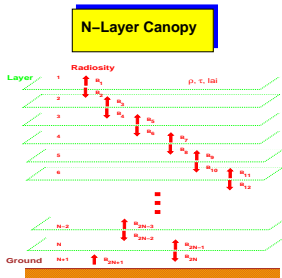


Figure 2: Radiosity of a N-canopy layer model above ground.

Table 1: Band limits and soil reflectances used

Channel Number	Lower Band limit in $\mu m$	Upper Band limit in $\mu m$	$\rho_{clay}$	$\rho_{sand}$	$\rho_{peat}$
1	0.45	0.52	0.417	0.336	0.096
2	0.52	0.60	0.466	0.382	0.097
3	0.63	0.69	0.524	0.415	0.146
4	0.76	0.90	0.576	0.447	0.280
5	1.55	1.75	0.646	0.500	0.546
7	2.08	2.35	0.633	0.451	0.452

### 6 Landsat TM channels:

#### Soil Reflectances:

SOILSPEC for  $\theta_s = 30^\circ$

#### Soil Types:

clay, sand and peat for a medium wet soil with smooth surface.

Soil reflectance  $\rho_s$ :

$$\rho_s = f_{clay} \rho_{clay} + f_{sand} \rho_{sand} + f_{peat} \rho_{peat}$$

where  $f_{clay} + f_{sand} + f_{peat} = 1$ .

#### Leaf Reflectances/Transmittances:

##### PROSPECT model

##### Parameters:

Uniform random distributed parameters between the  $\rho_{min}$  and  $\rho_{max}$

- Leaf structure  $N$ :  $N_{min} = 2$ ,  $N_{max} = 3$ .

- Chlorophyll pigment concentration:  $C_{chl}$  in  $[\mu gcm^{-2}]$ :  $C_{chl,min} = 7.85$ ,  $C_{chl,max} = 34.24$

- Water content:  $C_w$  in  $[g\%]$ :  $C_{w,min} = 0.008$ ,  $C_{w,max} = 0.014$

### Notes:

- Leaves with large  $N$  and small  $C_{chl}$  and small  $C_w$  are typical for senescent (yellow) leaves.
- Large chlorophyll content indicates a healthy green leaf.
- Single layer LAI from 0 to 1.
- 20-Layer model with LAI from 1 to 5.

## LAI Retrieval using Vegetation Indices

### Vegetation Indices (VIs):

1. Simple ratio index:

$$VI = \frac{nir}{red}$$

2. Normalized difference VI:

$$NDVI = \frac{nir - red}{nir + red}$$

3. Weighted difference VI:

$$WDVI = nir - a \cdot red$$

where

$$a = \frac{\rho_s(nir)}{\rho_s(red)}$$

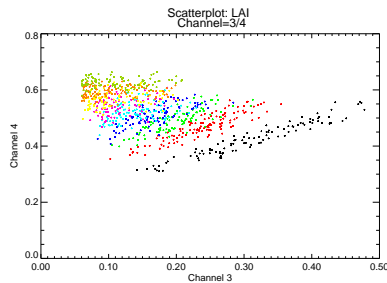


Figure 3: Scatterplot of canopy reflectances in TM-3 (red) and TM-4 (NIR) for a single canopy layer model above ground.

4. Soil adjusted VI (original):

$$SAVI = \frac{3}{2} \frac{nir - red}{nir + red + 0.5}$$

5. Soil adjusted VI (1):

$$SAVI1 = (1 + L) \frac{nir - red}{nir + red + L}$$

6. Soil adjusted VI (2):

$$SAVI2 = nir + 0.5 - \sqrt{(nir + 0.5)^2 - 2(nir - red)}$$

7. Nonlinear VI:

$$GEMI = \frac{\eta(1 - .25\eta) - (red - 0.125)}{1 - red}$$

where

$$\eta = \frac{2(nir^2 - red^2) + 1.2nir + 0.5red}{nir + red + 0.5}$$

8. Nonlinear VI:

$$NLI = \frac{nir^2 - red}{nir^2 + red}$$

where  $\bar{x}$  denotes an average over  $x$ . The band averaged reflectances in TM-3 and TM-4 channels were used as red and nir.

### Signal-to-Noise concept:

$$SNR(VI) = \frac{\max(VI_{i-1:N}) - \min(VI_{i-1:N})}{2\sigma(VI)_i} \quad (6)$$

where  $\overline{VI}_i$  denotes the average of  $VI$  in the  $i$ -th interval of the parameter of interest (e.g., if LAI varies from 0 to 1, and  $N = 10$  then the 5-th interval is from 0.5 to 0.6).  $\sigma(VI)_i$  is the standard deviation of  $VI$  for all VIs in the  $i$ -th interval of the parameter of interest.

### Interpretation of SNR:

- $SNR \leq 1 \rightarrow$  impossible to retrieve a parameter
- $SNR > 2 \rightarrow$  possible to retrieve a parameter

### Result:

GEMI index by Pinty and Verstraete (1992) performs very well ( $SNR > 5$ )

## Retrieval of Canopy Chemistry Parameters

## Principal Components/Neural Network Clustering Approach

### Assumption:

To retrieve of canopy chemistry more than two spectral bands are necessary

1. Compute the principal components of the 6 Landsat TM channel reflectances:  $\chi_i$ ,  $i = 1, 2, 3, 4, 5, 6$ .
2. Based on the values of the Eigenvectors choose 2 principal components:  $\chi_1$  and  $\chi_2$ .
3. The canopy spectra are projected on the 2 principal components  $\chi_1$  and  $\chi_2$ .
4. A neural-network (NN) is trained using all pairs of points and calculates weights using a learning algorithm.
5. The neural-network clusterer is used to classify each spectrum.

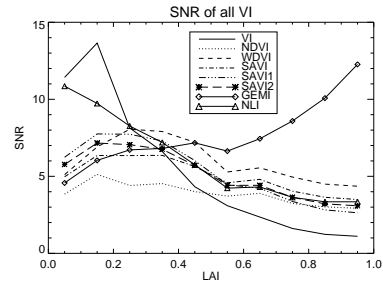


Figure 4: Signal-to-noise ratio of various VIs for a single layer canopy.

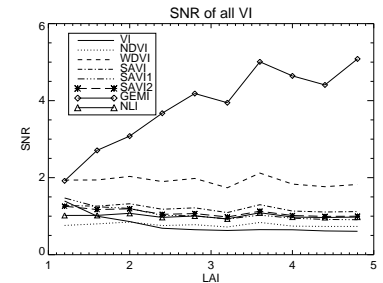


Figure 5: Signal-to-noise ratio of various VIs for a 20-layer canopy.

6. For each class compute mean and standard deviation of the parameters of interest and sort them in ascending order and graph them.

### Results:

1. The LAI of a single layer could be retrieved well and for the 20-layer model up to a LAI of about 2.
2. The leaf structure parameter was not retrievable for both canopy types.
3. The leaf water content can be distinguished for dense canopies but more or less just in senescent wet canopies.
4. The chlorophyll concentration was retrievable for dense canopies but not significant for single layer canopies.
5. The fractions of soil could not be estimated with this method.

## Geometry Based Approaches

### Features:

- TM(1,2,3) to retrieve the chlorophyll absorption
- TM(2,3,4) to retrieve the chlorophyll absorption
- TM(4,5,7) to retrieve leaf water content

### Distance/Triangle Approach:

1. The area  $A$  of the triangle formed by the points ( $W_1$ ,  $R_1$ ), ( $W_2$ ,  $R_2$ ) and ( $W_3$ ,  $R_3$ ).
2. The perpendicular distance  $d$  of the middle point ( $W_2$ ,  $R_2$ ) from a line described by the points between the minimum ( $W_1$ ,  $R_1$ ) and maximum ( $W_3$ ,  $R_3$ ) wavelength.

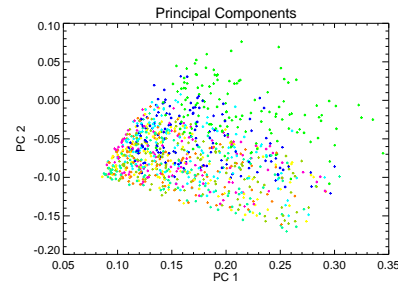


Figure 6: Example of scatterplot of first two principal components (colors indicate different LAIs).

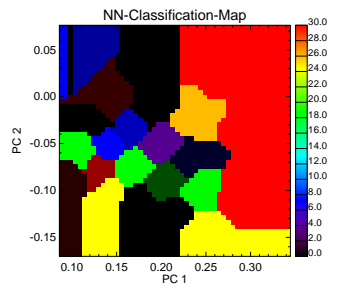


Figure 7: Example of a neural network classification map for 30 classes.

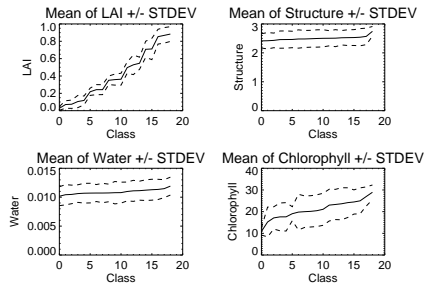


Figure 8: Mean and standard deviation of classes: a)  $LAI$ , b)  $N$ , c)  $C_w$  and d)  $C_{gr15}$  found with a neural network clustering algorithm for a single N-layer canopy.

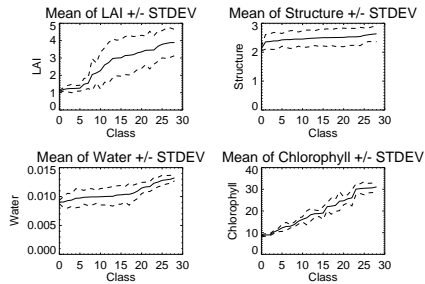


Figure 9: Mean and standard deviation of classes: a)  $LAI$ , b)  $N$ , c)  $C_w$  and d)  $C_{gr15}$  found with a neural network clustering algorithm for N-layer canopy.

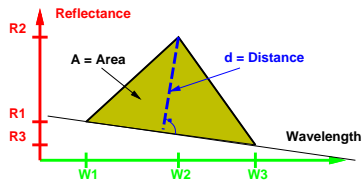


Figure 10: Geometry of the distance and area measures that relate chlorophyll concentration to the green peak of vegetation reflectance.

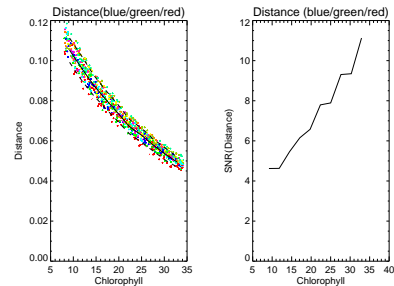


Figure 11: Scatterplot of the distance measure for the green peak as a function of chlorophyll concentration and the SNR measure for its retrieval.

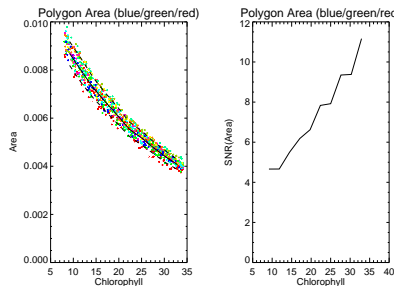


Figure 12: Scatterplot of the area measure for the green peak as a function of chlorophyll concentration and the SNR measure for its retrieval.

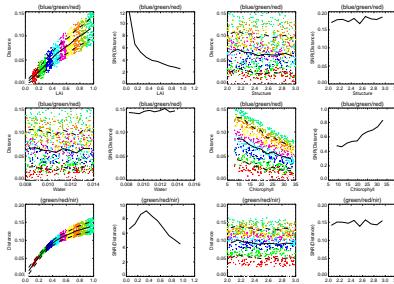


Figure 13: Scatterplot of the distance measure as a function of canopy parameter and the SNR measure for its retrieval (Single Layer) (colors indicate different LAIs).

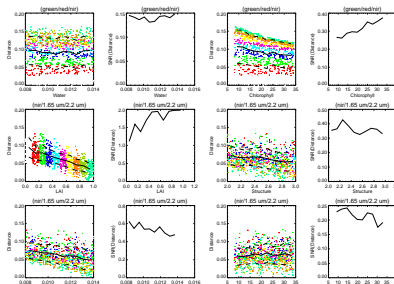


Figure 14: Scatterplot of the distance measure as a function of canopy parameter and the SNR measure for its retrieval (Single Layer) (colors indicate different LAIs).

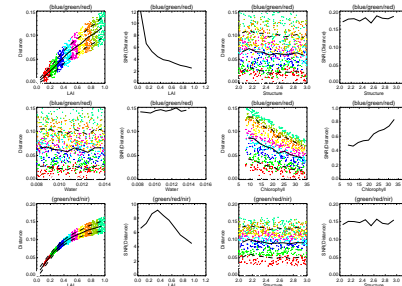


Figure 15: Scatterplot of the area measure as a function of chlorophyll concentration and the SNR measure for its retrieval (Single Layer) (colors indicate different LAIs).

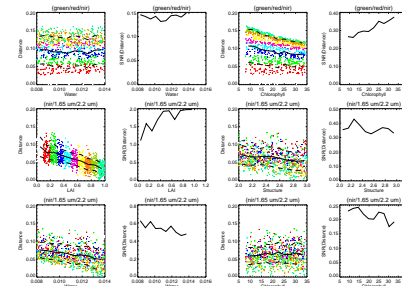


Figure 16: Scatterplot of the area measure as a function of chlorophyll concentration and the SNR measure for its retrieval (Single Layer) (colors indicate different LAIs).

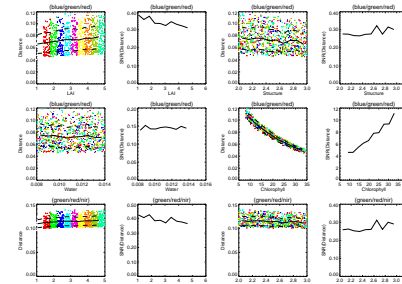


Figure 17: Scatterplot of the distance measure as a function of canopy parameter and the SNR measure for its retrieval (N-Layer).

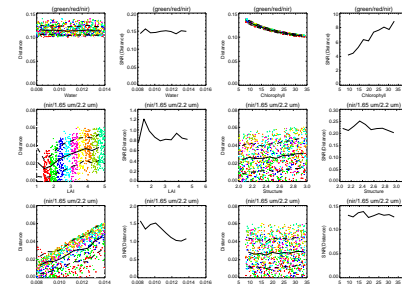


Figure 18: Scatterplot of the area measure as a function of canopy parameter and the SNR measure for its retrieval (N-Layer).

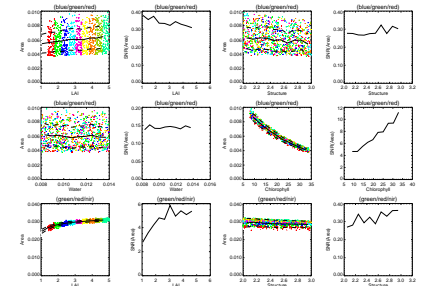


Figure 19: Scatterplot of the area measure as a function of chlorophyll concentration and the SNR measure for its retrieval (N-Layer).

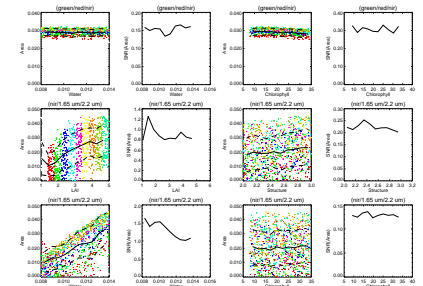


Figure 20: Scatterplot of the area measure as a function of chlorophyll concentration and the SNR measure for its retrieval (N-Layer).

## Conclusions

- Radiosity method together with physical models for leaf reflectance/transmittance (PROSPECT) and soil reflectance (SOILSPEC) can be used to generate datasets for multi-spectral sensors such as Landsat.
- Vegetation indices (VI, NDVI, WDI, SAV, SAVI, SAV2, GEMI and NLI) can retrieve the leaf area index.
- The GEMI index proved to perform best for single and N-layer canopies.
- More sophisticated algorithms based on principal components and neural network clustering were used to successfully retrieve LAI for sparse canopies.
- Geometric measures (distance and area) retrieve chlorophyll concentrations for dense canopies given three bands (TM-1, TM-2 and TM-3).

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